A propositions-as-types approach to the generalized crossover effect

Daiki Matsuoka[1] Daisuke Bekki[2] Hitomi Yanaka[1]
[1]The University of Tokyo [2]Ochanomizu University

Overview The *weak crossover* (WCO) effect is a constraint on anaphora that prohibits a quantifier from binding a pronoun that it does not c-command, as exemplified by (1).

(1) a. A girl^1 praised her$_1$ mother.

b. *Her$_1$ mother praised a girl^1.

Although WCO has traditionally been regarded as a restriction on syntactic movement, it has recently been observed that presupposition projection follows a similar constraint [1]. This general version of WCO, called *generalized crossover* (GCO), requires a uniform explanation of how quantifier scope and presupposition feed pronominal anaphora. To address this challenge, we adopt *Dependent Type Semantics* (DTS) [2], a semantic framework that uses a type as the semantic representation (SR) for a sentence. Under this view of *propositions-as-types*, semantic interactions among quantifier scope, anaphora, and presupposition can all be analyzed in terms of *scopal relations between types*, based on which GCO can be systematically derived.

GCO effect Anaphoric expressions have access to presuppositions that project out of presupposition holes (e.g., negation). This is illustrated by (2a), where the complement of *know* projects out of negation and is accessible from the pronoun *it*. However, this type of dependency is disallowed when the pronoun precedes the trigger, as in (2b).

(2) a. Alex did not **know** that Kim wrote a paper^1, and reviewed it$_1$.

b. *Its$_1$ reviewer did not **know** that Kim wrote a paper^1.

[1] suggested that the contrast in (2) should be considered parallel to the one attributed to WCO, proposing GCO, a generalization to the following effect.

(3) Quantifier scope (resp. presupposition) can feed a semantic dependency unless the semantically dependent expression "precedes" the quantifier (resp. the presupposition trigger).

As [1] pointed out, the standard movement-based analyses of WCO (e.g., [3]) would have difficulty providing a unified account for GCO, since they would need to posit separate mechanisms of binding for quantifier scope and presupposition. Hence, what we desire is a structural relation that governs the two phenomena and realizes GCO as a single constraint. We argue that this desideratum can be achieved by the propositions-as-types approach, which we will see next.

Framework DTS is based on the principle of propositions-as-types, which states that a proposition (and its proofs) can be identified with a type (and its terms) [4]. For instance, existential quantification $\exists x \in A.B$ is represented by the dependent product type $(x : A) \times B$, which consists of pairs $\langle a, b \rangle$ with $a : A$ and $b : B[x := a]$. When x does not occur free in B, the type corresponds to $A \wedge B$. For instance, *Kim wrote a paper* is translated into the SR (4). Note that $\texttt{paper}(x)$ takes scope as well as the existential quantification. As we will see, this parallelism between propositions and quantifier scopes is crucial in our analysis of GCO.

(4) $(x : \texttt{e}) \times ((u : \texttt{paper}(x)) \times \texttt{write}(\texttt{k}, x))$ (Let this SR be A_k)

DTS represents pronouns with the *underspecified type* $(x \mathbin{@} A) \times B$ (as illustrated in (5)), where x is a placeholder to be filled with a concrete term of type A. It handles presupposition triggers in the same way (6), in line with the presupposition-as-anaphora paradigm [5].

(5) Kim wrote a paper and Alex reviewed <u>it</u>. $\rightsquigarrow$ SR: $(v : A_k) \times (\underline{(y \mathbin{@} \texttt{e})} \times \texttt{review}(\texttt{a}, y))$

(6) Alex <u>knew</u> that Kim wrote a paper. $\rightsquigarrow$ SR: $\underline{(v \mathbin{@} A_k)} \times \texttt{know}(\texttt{a}, A_k)$

After these "intermediate" SRs are composed, underspecified types are eliminated in the process of *type checking*, thereby validating that an SR A is well-formed as a type under the current context Γ (formally, $\Gamma \vdash A : \mathsf{type}$). Figure 1 shows how this process works with (5) (to save space, we use the square-bracket notation $[\cdots]$ for types of the form $(\cdots) \times \cdots$). The type-checking algorithm inspects each part of an SR in a top-down manner (i.e., from higher scope to lower), and once it finds $(x \mathbin{@} A) \times \cdots$, it tries to construct a term of type A based on the contextual information, as depicted in the bottom-center box. Here, a possible result is $\pi_1 v : \mathtt{e}$, where π_1 is the function taking the first element a of a pair $\langle a, b\rangle$. Thus, the term $\pi_1 v$ refers to the entity x introduced in the first conjunct A_k, as shown by the annotated arrow in the resultant SR. Hence, we can correctly predict that *it* can covary with *a paper* in (5).

Proposal Although DTS provides a promising approach to pronominal binding, it cannot be applied as is to GCO. This leads us to propose the following two assumptions.

First, for a syntax-semantics interface providing proper treatment of inverse scope, we adopt the continuation-based grammar proposed by [6]. We stipulate that the LIFT operation is applied at the level of the lexicon, with pronouns (and presupposition triggers) being restricted to two-level towers. Informally, inverse scope is enabled by towers with more than two levels (Figure 2 (left)). Hence, the restriction here prevents a pronoun from being subject to inverse scope (Figure 2 (right)). In this setup, (1a) and (1b) are translated as shown on the right. Because of the top-down nature of type checking, x is available for $y \mathbin{@} \mathtt{e}$ in the SR for (1a) but not in the one for (1b). This scopal asymmetry leads to the correct prediction of WCO (although the same SRs are proposed in [2], we will argue in the full paper that the syntactic formalism adopted there can produce incorrect SRs).

SR for (1a):

$$\begin{bmatrix} \boxed{x : \mathtt{e}} \\ \begin{bmatrix} u : \mathtt{girl}(x) \\ \begin{bmatrix} \boxed{y \mathbin{@} \mathtt{e}} \\ \mathtt{praise}(x, \mathtt{mother}(y)) \end{bmatrix} \end{bmatrix} \end{bmatrix}$$

SR for (1b):

$$\begin{bmatrix} \boxed{y \mathbin{@} \mathtt{e}} \\ \begin{bmatrix} \boxed{x : \mathtt{e}} \\ \begin{bmatrix} u : \mathtt{girl}(x) \\ \mathtt{praise}(\mathtt{mother}(y), x) \end{bmatrix} \end{bmatrix} \end{bmatrix}$$

Second, to formalize presupposition projection in terms of type checking, we define the operation of *accommodation* as in (7), which extends the context upon presupposition failure. To illustrate, consider the negated version of (6). If the initial context does not entail A_k, type checking proceeds as described on the right. The result is that the accommodated presupposition A_k is not in the scope of negation, as expected.

Type checking of ¬(6):

$$\Gamma \vdash \neg \begin{bmatrix} v \mathbin{@} A_k \\ \mathtt{know}(\mathtt{a}, A_k) \end{bmatrix} : \mathsf{type}$$

$$\Big\downarrow \text{ type checking}$$

$$\Gamma, v : A_k \vdash \neg\mathtt{know}(\mathtt{a}, A_k) : \mathsf{type}$$

(7) Accommodation: if the type-checking algorithm cannot find any term M such that $\Gamma \vdash M : A$ in deriving $\Gamma \vdash (x \mathbin{@} A) \times B : \mathsf{type}$, then it can replace the result of this type checking with that of $\Gamma, x : A \vdash B : \mathsf{type}$.

Analysis We show the SRs for (2a) and (2b) on the right, and the type-checking process of the first one in Figure 3. Suppose that the initial context Γ does not entail A_k (if it does, binding should be possible in (2b), too). When the upper part $w : \neg[\cdots]$ is checked, presupposition failure occurs and A_k is accommodated. Then, the lower part is checked under the context including $v : A_k$ (see the bottom-right box), so y can be replaced with $\pi_1 v$, as in Figure 1. In the resultant SR, the projected presupposition feeds anaphora, as desired. As to the SR for (2b), $y \mathbin{@} \mathtt{e}$ must be eliminated before A_k is accommodated, meaning that v cannot be used to eliminate y. This explains the unavailability of binding in (2b).

SR for (2a):

$$\begin{bmatrix} w : \neg \begin{bmatrix} v \mathbin{@} A_k \\ \mathtt{know}(\mathtt{a}, A_k) \end{bmatrix} \\ \begin{bmatrix} y \mathbin{@} \mathtt{e} \\ \mathtt{review}(\mathtt{a}, y) \end{bmatrix} \end{bmatrix}$$

SR for (2b):

$$\begin{bmatrix} y \mathbin{@} \mathtt{e} \\ \neg \begin{bmatrix} v \mathbin{@} A_k \\ \mathtt{know}(\mathtt{rev}(y), A_k) \end{bmatrix} \end{bmatrix}$$

Conclusion With the DTS framework and the continuation-based syntax-semantics interface, the SRs for the GCO cases have the structurally parallel configurations shown below, where $>$ indicates the scopal relation between types. As a result, the GCO effect is uniformly predicted by the scope sensitivity of the anaphora resolution and the (proposed) accommodation operation performed during type checking.

(dependent exp.) $\cdots$ quantifier	(dependent exp.) $\cdots$ [operator $\cdots$ (trigger) $\cdots$]
$\rightsquigarrow$ SR: $(y \mathbin{@} B) > (x : A)$	$\rightsquigarrow$ SR: $(y \mathbin{@} B) > Op(\cdots (x \mathbin{@} A) \cdots)$

References **[1]** P. D. Elliott and Y. Sudo. Generalised crossover. In *Semantics and Linguistic Theory*, volume 30, pages 396–408, 2021. **[2]** D. Bekki. A proof-theoretic analysis of weak crossover. In *New Frontiers in Artificial Intelligence: JSAI-isAI 2021 Workshops, JURISIN, LENLS18, SCIDOCA, Kansei-AI, AI-BIZ, Yokohama, Japan, November 13–15, 2021, Revised Selected Papers*, pages 228–241. Springer Nature Switzerland, 2023. **[3]** T. Reinhart. *Anaphora and Semantic Interpretation*. Routledge, 1983. **[4]** P. Dybjer and E. Palmgren. Intuitionistic Type Theory. In *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, spring 2023 edition, 2023. **[5]** R. van der Sandt. Presupposition Projection as Anaphora Resolution. *Journal of Semantics*, 9(4):333–377, 1992. **[6]** C. Barker and C. Shan. *Continuations and Natural Language*, volume 53 of *Oxford studies in theoretical linguistics*. Oxford University Press, 2014.

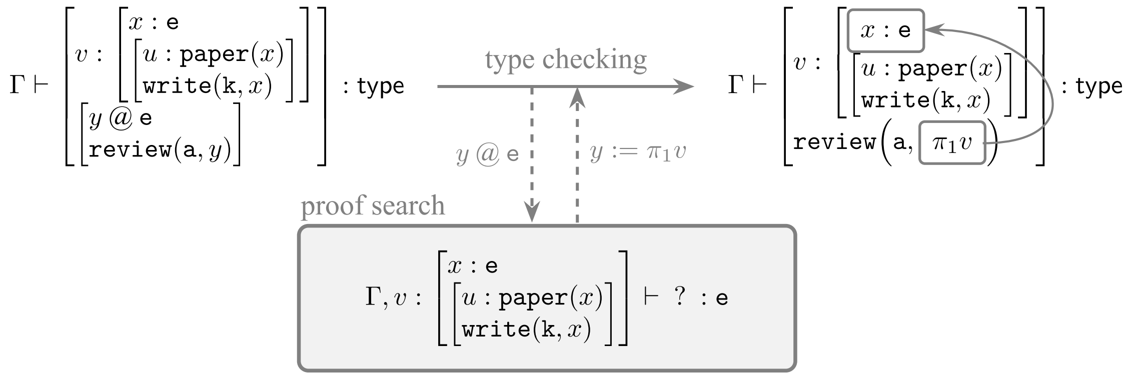

Figure 1: Type checking of the SR for (5). Note that $\begin{bmatrix} x : A \\ B \end{bmatrix}$ stands for $(x : A) \times B$ (the same applies to @).

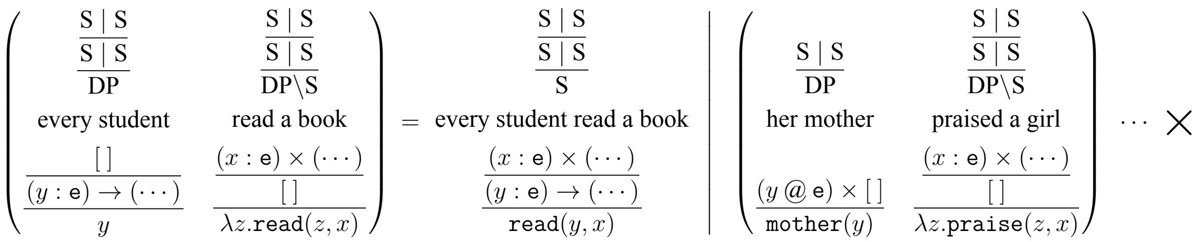

Figure 2: Left: derivation of *every student read a book* with the inverse scope reading (see [6] for the definition of the *tower notation*). Right: attempt to derive (1b) forcing the indefinite to take higher scope than the pronoun, which fails due to the mismatch of the tower levels.

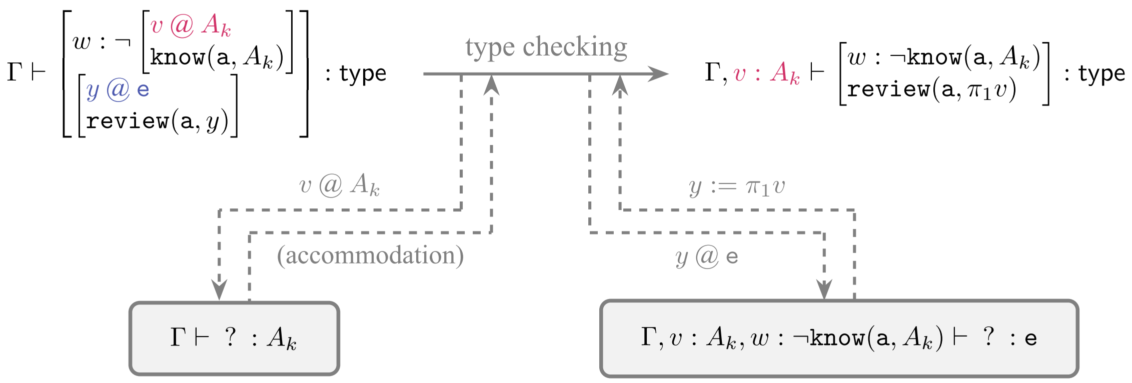

Figure 3: Type checking of the SR for (2a).